

# Bestimmung der spezifischen Elektroenenladung (e/m)

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Group B412

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## 1. EINLEITUNG

In this paper the specific electron charge is examined. The experiments took place on the 13<sup>th</sup> of August 2009 in the department for Mathematic and Computer Science, room number: 0.04.038.

## 2. BESCHREIBUNG DER VERWENDETEN METHODE UND VERSUCHSAUFBAU

Moving electrons are deflected in a magnetic field according to the Lorentz-Law. In this experiment a Helmholtz coil is used as it provides a stable magnetic field. By measuring the amount they are deflected in a certain magnetic field one can obtain a value for the specific electron charge.

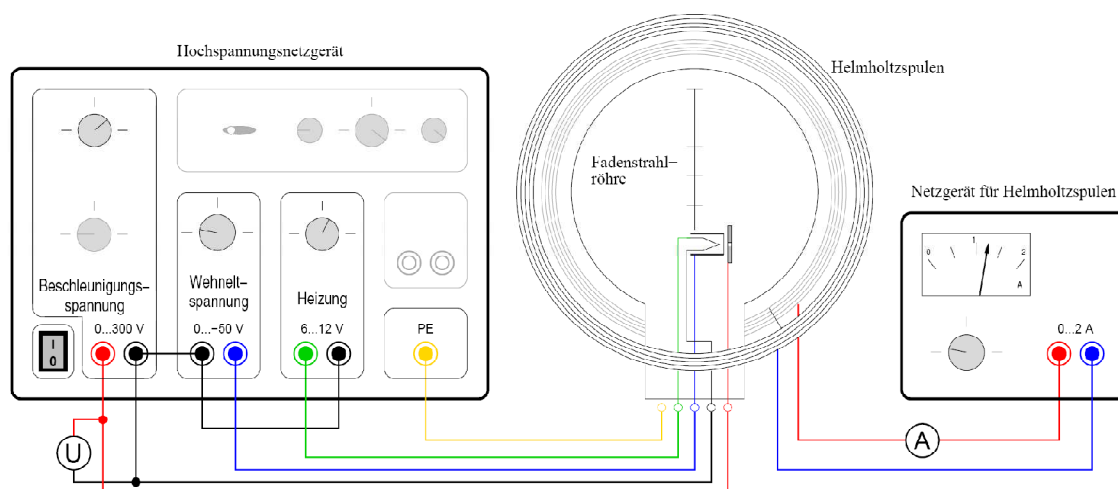


FIGURE 1: EXPERIMENTAL SETUP

## 3. EXPERIMENTELLE DURCHFÜHRUNG

The electrons (mass  $m_e$ , charge  $q$ ) emitted by a heating wire are accelerated with a certain voltage  $U_B$  and enter the magnetic field  $B$  of the Helmholtz coil. In the first series of experiments the magnetic field is kept constant and the acceleration voltage is adjusted each time in order to adjust the trajectory of the electrons to form a perfect circle with the radiuses  $r = 30$  mm,  $40$  mm and  $50$  mm. In the second series of experiments the acceleration voltage is kept constant and the magnetic field is adjusted for every single radius.

To determine the specific electron charge the following formulas are used:

$$T = \frac{1}{2} m_e v_B^2 = q \cdot U_B \quad (1)$$

$$v_B = \sqrt{\frac{2qU_B}{m_e}} \quad (2)$$

This formula is only valid for non relativistic speeds, in our case the maximum acceleration voltage was  $300$  V, thus the highest velocity  $v_b$  that can be reached is:  $0,034$  c. Consequently it is not necessary to calculate relativistically.

Another aspect in the error calculations is that the electrons have an initial velocity  $v_T$  as they are emitted by a heating wire. In formula (2) this is not included. In our case the mean temperature  $T$  of the heating wire was  $1000^\circ\text{C}$ .

$$v_{\text{initial}} = \sqrt{\frac{3k_B T}{m_e}} \quad (k_B = \text{Boltzmannconstant}) \quad (3)$$

Temperature of the heating wire [k]	Thermal velocity $v_T$ [m/s]	Velocity $v_B$ according to formula (2) with $U_B = 100\text{V}$	$\frac{v_T}{v_B}$ [in %]
77,4	$5,93 \cdot 10^4$	$5,93 \cdot 10^6$	1,0%
1273,2	$2,41 \cdot 10^5$	$5,93 \cdot 10^6$	4,1%

TABLE 1: THERMAL VELOCITY CALCULATIONS

The table above shows that the effect is significant. In the following calculations it is ignored but will be considered in the sources of errors.

#### Determination of $q/m$ :

For moving electrons in a magnetic field the forces are balanced: The Lorentz force equals the centripetal force:

$$|F_{\text{Lorentz}}| = |q| \cdot v_B \cdot B = \frac{m_e \cdot v_B^2}{r} = |F_{\text{Centripetal}}| \quad (4)$$

The magnetic field in the center of the Helmholtz coil ( $N = 130$ ,  $R = 0,15$  m) can be approximated with the formula:

$$B = \mu_0 \cdot \left(\frac{4}{5}\right)^{3/2} \cdot N \cdot \frac{I}{R} \quad (\mu_0 = \text{magnetic constant}) \quad (5)$$

The only changeable value is the coil current  $I$ .

By solving formula (4) the specific electron charge is:

$$\frac{q}{m} = \frac{2 \cdot U_B}{B^2 \cdot r^2} \quad (6)$$

## 4. AUSWERTUNG

### 4.1 ANALYTISCHE AUSWERTUNG

Data on page: 6; Error calculation on page: 6ff.

#### 1. Measurements with a constant magnetic field: $B = (1,01 \pm 0,02) \cdot 10^{-3}$ T

Radius $r$ [m]	$0,0500 \pm 0,0015$	$0,0400 \pm 0,0015$	$0,0300 \pm 0,0015$
$U_B$ mean [V]	$219,0 \pm 0,7$	$143,0 \pm 0,2$	$84,0 \pm 0$
$\frac{ q }{m} \left[ \frac{As}{kg} \right]$	$(1,71 \pm 0,15) \cdot 10^{11}$	$(1,74 \pm 0,16) \cdot 10^{11}$	$(1,82 \pm 0,18) \cdot 10^{11}$

TABLE 2: MEASUREMENTS WITH A CONSTANT MAGNETIC FIELD

#### 2. Measurements with a constant acceleration voltage: $U_B = (130 \pm 1)V$

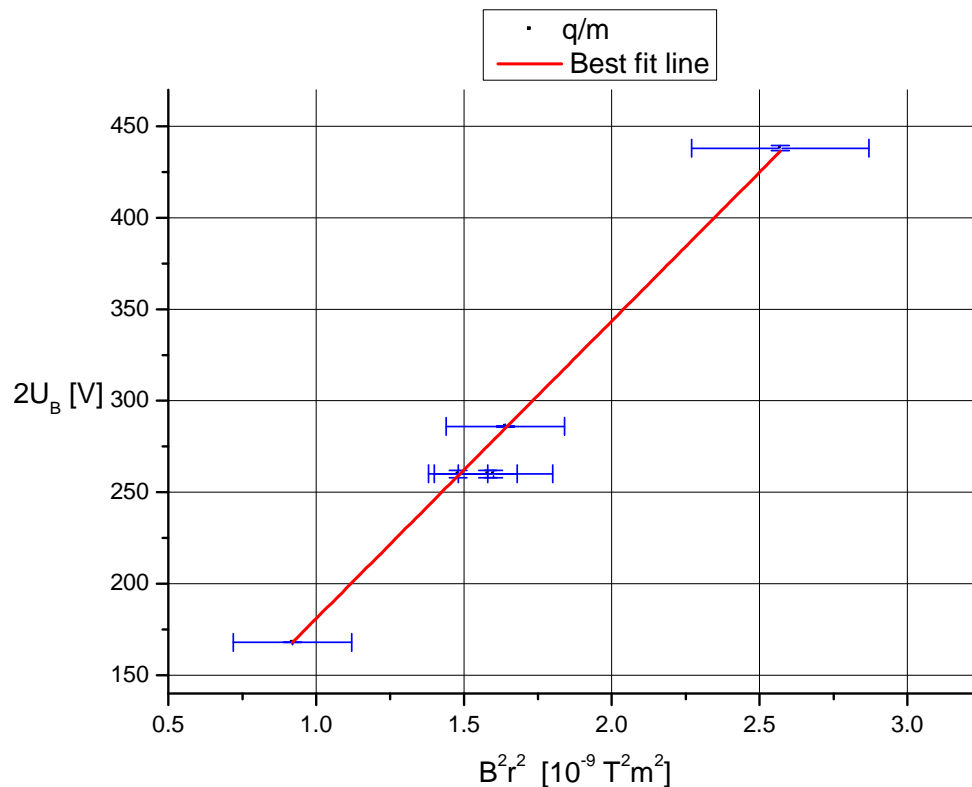
Radius $r$ [m]	$0,0500 \pm 0,0015$	$0,0400 \pm 0,0015$	$0,0300 \pm 0,0015$
$I$ mean [V]	$1,019 \pm 0,001$	$1,235 \pm 0,002$	$1,713 \pm 0,002$
$\frac{ q }{m} \left[ \frac{As}{kg} \right]$	$(1,65 \pm 0,14) \cdot 10^{11}$	$(1,75 \pm 0,15) \cdot 10^{11}$	$(1,62 \pm 0,20) \cdot 10^{11}$

TABLE 3: MEASUREMENTS WITH A CONSTANT ACCELERATION VOLTAGE

Total result with these six values:  $\left(\frac{|q|}{m}\right)_{mean} = (1,72 \pm 0,12) \cdot 10^{11} \frac{As}{kg}$ ,

### 4.2 GRAPHISCHE AUSWERTUNG

Another way to determine the specific electron charge is the graphical analysis. In the following diagram the slope is  $\frac{2 \cdot U_B}{B^2 \cdot r^2}$  which is exactly  $\frac{q}{m}$ .



GRAPH 1: Q/M WITH ERROR BARS AND BEST FIT LINE

As a result  $\frac{q}{m} = (1,62 \pm 0,03) \cdot 10^{11} \frac{\text{C}}{\text{kg}}$ . This is calculated with Origin, the value differs from the analytical one because in the graph the slope is loaded instrumentally, considering the quantity of the error.

## 5. DISKUSSION DER ERGEBNISSE

The determination of the specific electron charge with the Helmholtz Coil is a very good way. The received value in the analytical analysis differs only 2% from the literature value ( $1,76 \cdot 10^{11} \frac{\text{C}}{\text{kg}}$ ), whereas the graphical value differs 8% from the literature value.

There are a lot of statistical and systematic errors which are accounted for in the error calculation. One error that is not in the calculation is the heating effect of the wire. All the emitted electrons have different initial speeds, which results in the spreading of the beam. Its effect on the result is roughly 4%. But if you would consider this in the formulas (4) – (6), they would get very difficult.

Another source of error is the influence of the earth's magnetic field. This field is directed in a north south direction.

In our case the setup is aligned in a north - south direction. According to the right-hand rule the earth's magnetic field causes the electrons to form a spiral trajectory which has no effect on the radius of the circle. A further possible case is that the setup is aligned in a west - east direction. Consequently the radius decreases.

In our experiment the magnetic field of the Helmholtz coil was approximately 1 mT. The earth's magnetic field in Germany in the horizontal direction is about 20  $\mu\text{T}$  which causes a 2% deviation. The real radius would be 0,96 of the measured one. To prove this theory the radius is measured in each direction (north-south/west-east) with constant  $U_B$  and  $B_{\text{coil}}$ . There should be a difference of 1 – 3 mm. This difference can be detected, but only qualitatively, as it is very difficult to read the exact radius.

## 6. ZUSAMMENFASSUNG

This experiment shows how easy it actually is to measure something as small as the specific electron charge. Still a lot of errors occur as the environmental conditions are not perfect. With a bigger Helmholtz coil, a better way to measure the radius of the circle and the including of the initial speed of the electrons the result would get much more precise.

## 7. ANHANG

### 7.1 DATEN

- Measured Data in the experiment:

1st measurement with constnat magnetic field											
I [A]	1,3										
r [m]	0,05 B [T] 1,01E-03										
Trial	1	2	3	4	5	6	7	8	9	10	U Mean
Voltage	222	221	222	220	217	217	218	218	217	218	219
r [m]	0,04 B [T] 1,01E-03										
Trial	1	2	3	4	5	6	7	8	9	10	
Voltage	142	142	143	144	143	143	143	144	143	143	143
r [m]	0,03 B [T] 1,01E-03										
Trial	1	2	3	4	5	6	7	8	9	10	
Voltage	84	84	84	84	84	84	84	84	84	84	84
2nd measurement with constant acceleration voltage											
Ub [V]	130										
r [m]	0,05 B [T] 7,94E-04										
Trial	1	2	3	4	5	6	7	8	9	10	I Mean
Current	1,01	1,02	1,02	1,02	1,02	1,02	1,02	1,02	1,02	1,02	1,019
r [m]	0,04 B [T] 9,62E-04										
Trial	1	2	3	4	5	6	7	8	9	10	
Current	1,24	1,23	1,23	1,22	1,24	1,24	1,24	1,24	1,24	1,23	1,235
r [m]	0,03 B [T] 1,33E-03										
Trial	1	2	3	4	5	6	7	8	9	10	
Current	1,72	1,71	1,71	1,71	1,7	1,72	1,71	1,71	1,72	1,72	1,713

- Data for the graph

Equation	$y = a + b \cdot x$		
Loading	Instrumental		
		Value	Standard error
a	Intersection with the Y - axis	18,40	2,55
b	Slope	1,62E11	2,78

$B^2 r^2 (X)$	$2U_B (Y)$	Error bar X- dir.	Error bar Y -dir.
0,92E-09	168,0	0,2E-09	0
1,48E-09	260,0	0,1E-09	2,0
1,58E-09	260,0	0,1E-09	2,0
1,60E-09	260,0	0,2E-09	2,0
1,64E-09	286,0	0,2E-09	0,4
2,57E-09	438,0	0,3E-09	1,4

### 7.2 FEHLERRECHNUNG

#### 7.2.1 1. MESSUNG MIT KONSTANTEM MAGNETFELD

- Error of the magnetic field

$$B = \mu_0 \cdot \left(\frac{4}{5}\right)^{3/2} \cdot N \cdot \frac{I}{R} ; I = (1,30 \pm 0,01) \text{ A}; R = (0,150 \pm 0,002) \text{ m}$$

$$\Delta B_{stat} = \sqrt{\Delta I^2 \cdot \left[\frac{\partial B}{\partial I}\right]^2} = 7,8 \mu T$$

$$\Delta B_{sys} = \left| \Delta R \cdot \left[\frac{\partial B}{\partial R}\right] \right| = 13,5 \mu T$$

$$\Delta B = \Delta B_{stat} + \Delta B_{sys} = 21 \mu T$$

- Mean statistical deviation of the acceleration voltage:

$$\sigma_{U_B} = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^n (U_{Bi} - \bar{U}_B)^2}$$

$$\sigma_{\bar{U}_B} = \frac{1}{\sqrt{n}} \sigma_{U_B} = \text{see table below}$$

- Error of the specific electron charge

$$\frac{q}{m} = \frac{2 \cdot U_B}{B^2 \cdot r^2}$$

$$\Delta \left(\frac{q}{m}\right)_{stat} = \sqrt{\Delta U_B^2 \cdot \left[\frac{\partial \left(\frac{q}{m}\right)}{\partial U_B}\right]^2 + \Delta B_{stat}^2 \cdot \left[\frac{\partial \left(\frac{q}{m}\right)}{\partial B}\right]^2 + \Delta r_{stat}^2 \cdot \left[\frac{\partial \left(\frac{q}{m}\right)}{\partial r}\right]^2} = \text{see table below}$$

$$\Delta \left(\frac{q}{m}\right)_{sys} = \left| \Delta B_{sys} \cdot \left[\frac{\partial \left(\frac{q}{m}\right)}{\partial B_{sys}}\right] \right| + \left| \Delta r_{sys} \cdot \left[\frac{\partial \left(\frac{q}{m}\right)}{\partial r_{sys}}\right] \right| = \text{see table below}; (\Delta r_{sys} = \pm 0,0005 \text{ m})$$

$$\Delta \left(\frac{q}{m}\right) = \Delta \left(\frac{q}{m}\right)_{stat} + \Delta \left(\frac{q}{m}\right)_{sys} = \text{see table below}$$

- a)  $r = 0,050 \text{ m}$

Fehlertabelle:

$\sigma_{\bar{U}_B}$ (statistic)	$\pm 0,7 \text{ V}$
$\Delta r$ (Electron radius, statistic)	$\pm 0,001 \text{ m}$
$\Delta \left(\frac{q}{m}\right)_{stat}$	$\pm 7,4 \cdot 10^9 \frac{\text{C}}{\text{kg}}$
$\Delta \left(\frac{q}{m}\right)_{sys}$	$\pm 8,0 \cdot 10^9 \frac{\text{C}}{\text{kg}}$
$\Delta \left(\frac{q}{m}\right)$ (stat + sys)	$\pm 15 \cdot 10^9 \frac{\text{C}}{\text{kg}}$

- b)  $r = 0,040 \text{ m}$

Fehler:

$\sigma_{\bar{U}_B}$ (statisc)	$\pm 0,2 V$
$\Delta r$ (Electron radius, statistic)	$\pm 0,001 m$
$\Delta \left(\frac{q}{m}\right)_{stat}$	$\pm 7,4 \cdot 10^9 \frac{C}{kg}$
$\Delta \left(\frac{q}{m}\right)_{sys}$	$\pm 9,0 \cdot 10^9 \frac{C}{kg}$
$\Delta \left(\frac{q}{m}\right) (stat + sys)$	$\pm 16 \cdot 10^9 \frac{C}{kg}$

c)  $r = 0,030 m$ 

$\sigma_{\bar{U}_B}$ (statisc)	$\pm 0,0 V$
$\Delta r$ (Electron radius, statistic)	$\pm 0,001 m$
$\Delta \left(\frac{q}{m}\right)_{stat}$	$\pm 7,4 \cdot 10^9 \frac{C}{kg}$
$\Delta \left(\frac{q}{m}\right)_{sys}$	$\pm 10,7 \cdot 10^9 \frac{C}{kg}$
$\Delta \left(\frac{q}{m}\right) (stat + sys)$	$\pm 18 \cdot 10^9 \frac{C}{kg}$

**Error calculation 2nd measurement with constant acceleration voltage:  $U_B = (130 \pm 1) V$** 

- Mean statistical deviation of the current of the magnetic field

$$\sigma_I = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^n (I_i - \bar{I})^2}$$

$$\sigma_{\bar{I}} = \frac{1}{\sqrt{n}} \sigma_I = \text{see table below}$$

- Error of the magnetic field

$$B = \mu_0 \cdot \left(\frac{4}{5}\right)^{3/2} \cdot N \cdot \frac{I}{R} ; I = (1,30 \pm 0,01) A; R = (0,150 \pm 0,002) m$$

$$\Delta B_{stat} = \sqrt{\Delta I^2 \cdot \left[\frac{\partial B}{\partial I}\right]^2} = \text{see table below}$$

$$\Delta B_{sys} = \left| \Delta R \cdot \left[\frac{\partial B}{\partial R}\right] \right| = \text{see table below}$$

$$\Delta B = \Delta B_{stat} + \Delta B_{sys} = \text{see table below}$$

a)  $r = 0,050 m$ 

$\sigma_{\bar{I}}$ (statisc)	$\pm 0,001 A$
$\Delta r$ (Electron radius, statistic)	$\pm 0,001 m$
$\Delta B_{stat}$	$\pm 0,78 \mu T$

$\Delta B_{sys}$	$\pm 8,4 \mu T$
$\Delta B (stat + sys)$	$\pm 9 \mu T$
$\Delta \left(\frac{q}{m}\right)_{stat}$	$\pm 6,7 \cdot 10^9 \frac{C}{kg}$
$\Delta \left(\frac{q}{m}\right)_{sys}$	$\pm 6,8 \cdot 10^9 \frac{C}{kg}$
$\Delta \left(\frac{q}{m}\right) (stat + sys)$	$\pm 14 \cdot 10^9 \frac{C}{kg}$

b)  $r = 0,040 \text{ m}$ 

$\sigma_{\bar{I}} (statisc)$	$\pm 0,002 \text{ A}$
$\Delta r (Electron \text{ radius, statistic})$	$\pm 0,001 \text{ m}$
$\Delta B_{stat}$	$\pm 1,6 \mu T$
$\Delta B_{sys}$	$\pm 10 \mu T$
$\Delta B (stat + sys)$	$\pm 12 \mu T$
$\Delta \left(\frac{q}{m}\right)_{stat}$	$\pm 8,8 \cdot 10^9 \frac{C}{kg}$
$\Delta \left(\frac{q}{m}\right)_{sys}$	$\pm 6,2 \cdot 10^9 \frac{C}{kg}$
$\Delta \left(\frac{q}{m}\right) (stat + sys)$	$\pm 15 \cdot 10^9 \frac{C}{kg}$

c)  $r = 0,030 \text{ m}$ 

$\sigma_{\bar{I}} (statisc)$	$\pm 0,002 \text{ A}$
$\Delta r (Electron \text{ radius, statistic})$	$\pm 0,001 \text{ m}$
$\Delta B_{stat}$	$\pm 1,6 \mu T$
$\Delta B_{sys}$	$\pm 14 \mu T$
$\Delta B (stat + sys)$	$\pm 17 \mu T$
$\Delta \left(\frac{q}{m}\right)_{stat}$	$\pm 11 \cdot 10^9 \frac{C}{kg}$
$\Delta \left(\frac{q}{m}\right)_{sys}$	$\pm 8,7 \cdot 10^9 \frac{C}{kg}$
$\Delta \left(\frac{q}{m}\right) (stat + sys)$	$\pm 20 \cdot 10^9 \frac{C}{kg}$

- Error calculation of  $\left(\frac{q}{m}\right)_{mean}$  statistical:

$$\sigma_{\frac{q}{m}} = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^n \left(\frac{q}{m} - \bar{\frac{q}{m}}\right)^2} = 7,2 \cdot 10^9 \frac{C}{kg}$$

$$\sigma_{\bar{\frac{q}{m}}} = \frac{1}{\sqrt{n}} \sigma_{\frac{q}{m}} = 3,0 \cdot 10^9 \frac{C}{kg}$$

$$\text{Systematical Error of } \left(\frac{q}{m}\right)_{mean} = 8,2 \cdot 10^9 \frac{C}{kg}$$

## 8. QUELLEN

1. M. Saß. "Umgang mit Unsicherheiten". 09.07.08.

2. M. Saß. "Elektrodynamik und Magnetismus – Bestimmung von e/m". 05.09.08.