TECHNISCHE UNIVERSITÄT MÜNCHEN

Vacuum (VAK)

Group B412

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0. Content

1. Introduction

A real vacuum does not exist, but humanity is able to produce fairly good vacuums with the help of high-tech pressure pumps. In today's experiment we examine the pumping speed and the pumping time for different tubes of a specific pressure pump. In the following calculations it is necessary to assume ideal gas.

2. Description of the used methods

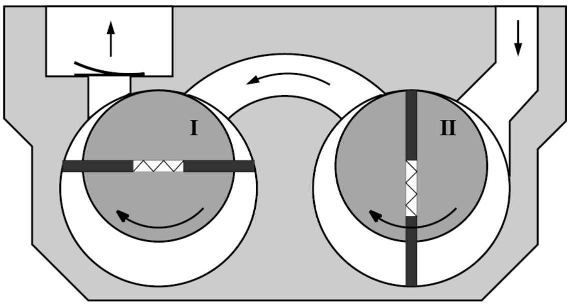


Figure 1: Cut-away of a rotary-vane pump

In the experiment a rotary-vane pump is used to evacuate the object of interest.

As seen in Figure 1 the two cylinders rotate constantly to transport the sucked in air towards the outlet on the left. Springs provide that the cylinders are pressed against the wall in order not to let the air flow back. Thus a pressure around $10^{-3} hPa$ or better can be reached.

3. Experimental Procedure

3.1 Calibration of the Pirani gauge

Before we start the experiment we need to calibrate the heat-conduction gauge.
The Pirani gauge which is used in the experiment uses the temperature dependency of the ohmic resistance in order to gauge the pressure. It adjusts the current through the wire to compensate the decrease in temperature caused by the pressure decrease. The aim is to have absolutely no current between left and the right side, so that the resistances are all the same: 50Ω. (see Wheatstone bridge figure to the right)
Consequently each pressure has got a unique current and the Pirani gauge can be calibrated by measuring the current and noting the corresponding pressure measured by an already calibrated manometer (vacuum gauge), see figure 3.

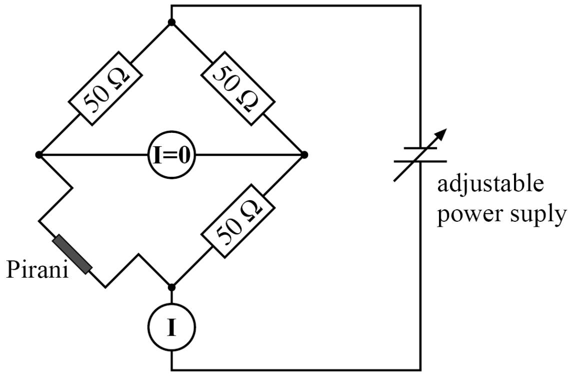


FIGURE 2: WHEATSTONE BRIDGE

For the calibration the following setup is needed:

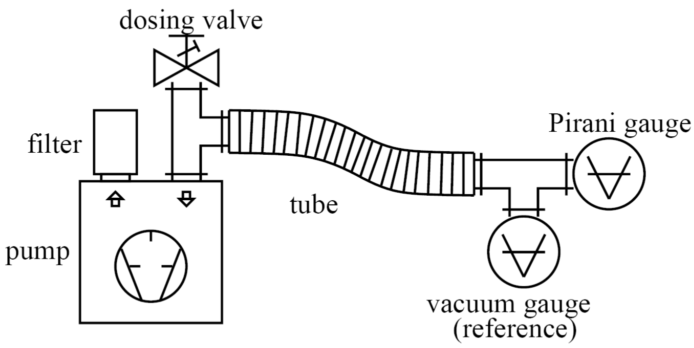
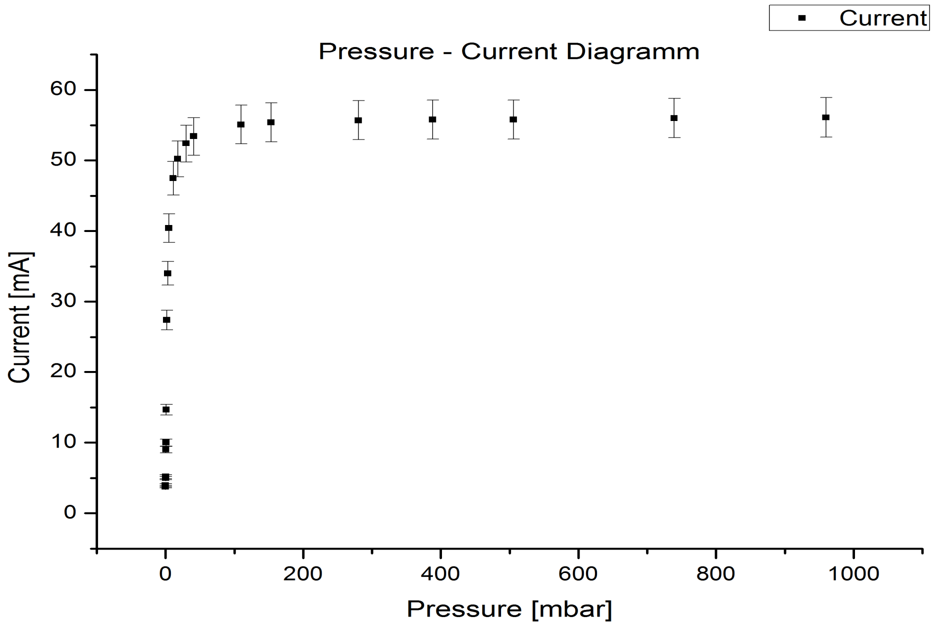


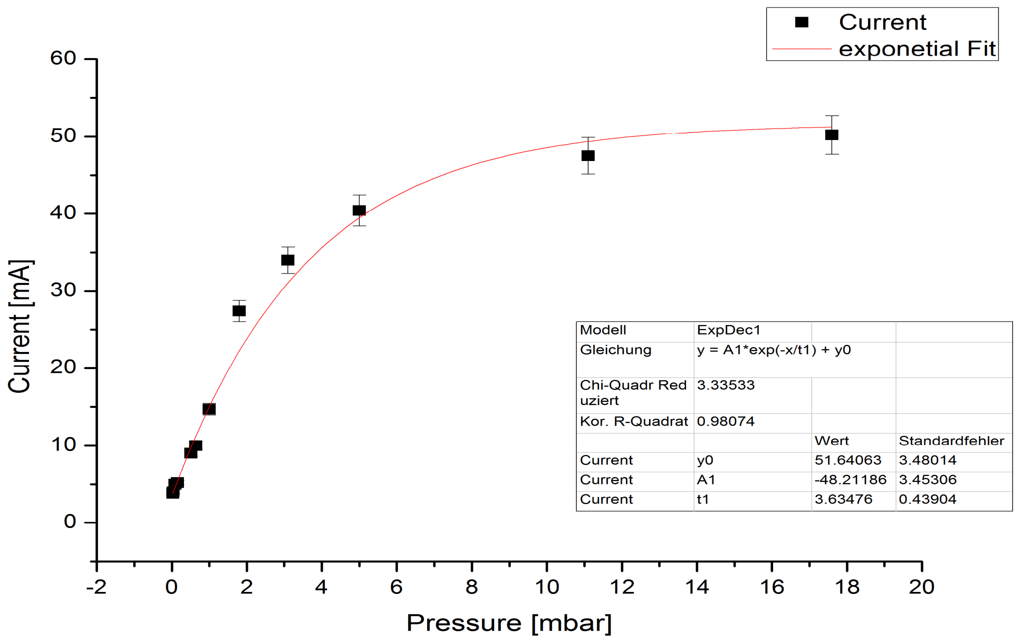
FIGURE 3: EXPERIMENTAL ARRANGEMENT FOR THE PIRANI-GAUGE CALIBRATION

With the received data for the current and the pressure it is possible to draw a "Pressure – Current – Diagram":



GRAPH 1: PRESSURE – CURRENT – DIAGRAM

As the gas does not behave ideal for high pressures, the calibration data is just taken of the low pressures. In the following diagram a best fit curve is plotted and used to create the calibration formula:

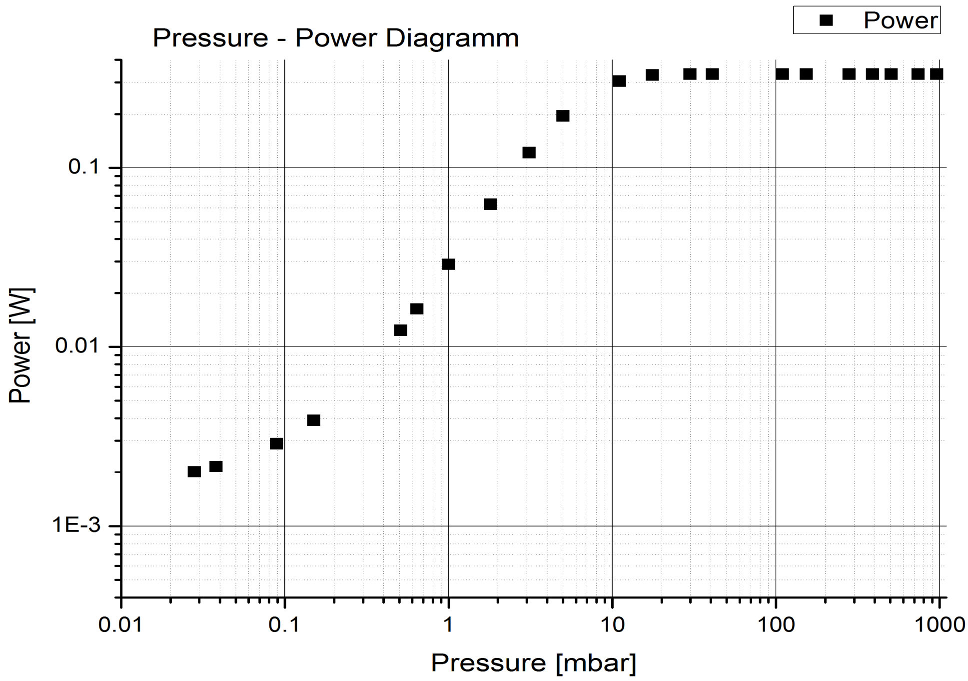


GRAPH 2: PRESSURE – CURRENT – DIAGRAM, LOW PRESSURE WITH BEST FIT CURVE

In the Figure 2 the potential correlation is obvious and can be written as:

$$I(t) = -48{,}21 \cdot e^{-\frac{p}{3{,}63}} \cdot A + 50{,}6 \cdot A$$

Furthermore it is useful to draw an electric power as a function of the pressure in a log-log-scale:



GRAPH 3: PRESSURE POWER DIAGRAM, DATA ON PAGE 11

The errors in this diagram are within the dot size. The error is different for each pressure and power. The errors in the following table are statistical and systematical errors of the instruments.

Pressure [mbar] interval	Error [mbar]	Power interval [W]	Error [W]
0,020 – 0,10	0,001	0,0010 – 0,0100	0,0005
0,10 – 1,00	0,01	0,010 – 0,100	0,005
1,0 -10,0	0,1	0,10 – 1,00	0,05
10 - 965	1		

3.2 Measuring of the pumping speed

After having successfully calibrated the Pirani gauge, it is now possible to examine some of the properties of the pump.

First of all the pumping speed S is measured in the following experiment.

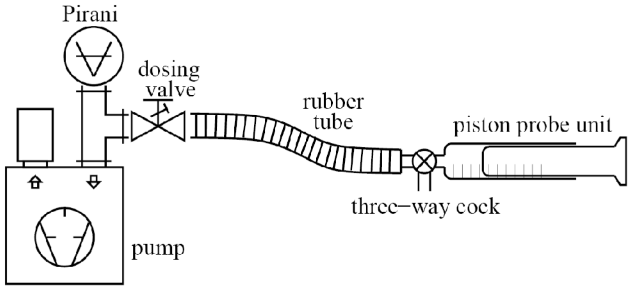


FIGURE 3: THE MEASUREMENT OF THE PUMPING SPEED

The pump is adjusted with the dosing valve to a certain constant pressure with the help of the heat-conduction gauge. In our case the current was $(26{,}5 \pm 0{,}2)\ mA$. With formula (1) the pressure p_0 at the pump inlet follows: $(0{,}75 \pm 0{,}01)\ mbar$.

The error is calculated with the following formula:

$$\Delta p_{stat} = \sqrt{\Delta I^2 \cdot \left[\frac{\partial p}{\partial I}\right]^2} = 0{,}01 \quad (2)$$

The pressure p in the probe unit is 1000 mbar. Then the time is measured it takes to empty a 100 ml piston probe unit, by reading off the time every 10 ml.
This procedure is performed three times and 30 values for S are obtained. Data on page: 11f. S is obtained with the following formula:

$$S_i = \frac{p}{p_0} \cdot \frac{dV}{dt} \quad (3)$$

➔ $S_{mean} = (2{,}98 \pm 0{,}02)\ \frac{m^3}{h}$

For the standard deviation of the pumping speed S one gets:

$$\sigma_S = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} (S_i - \bar{S})^2} = 0{,}12 \quad (4)$$

$$\sigma_{\bar{S}} = \frac{1}{\sqrt{n}} \sigma_S = 0{,}023 \quad (5)$$

This value differs a lot from the value given by the product specifications, $S = 3{,}7 \frac{m^3}{h}$. The difference can be explained by:

- The frictional force in the piston probe unit

The pressure p_0 was not the lowest possible pressure the pump can produce. By lowering the pressure p_0 in formula (3) S increases

3.3 Effective pumping speed with different tube sizes

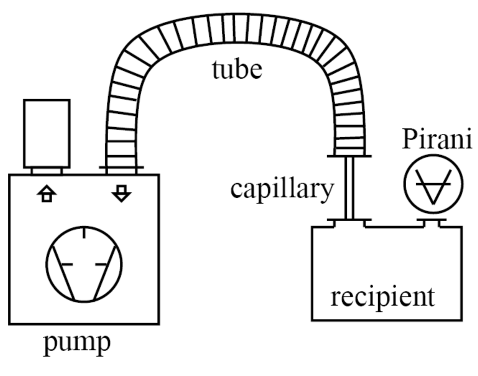
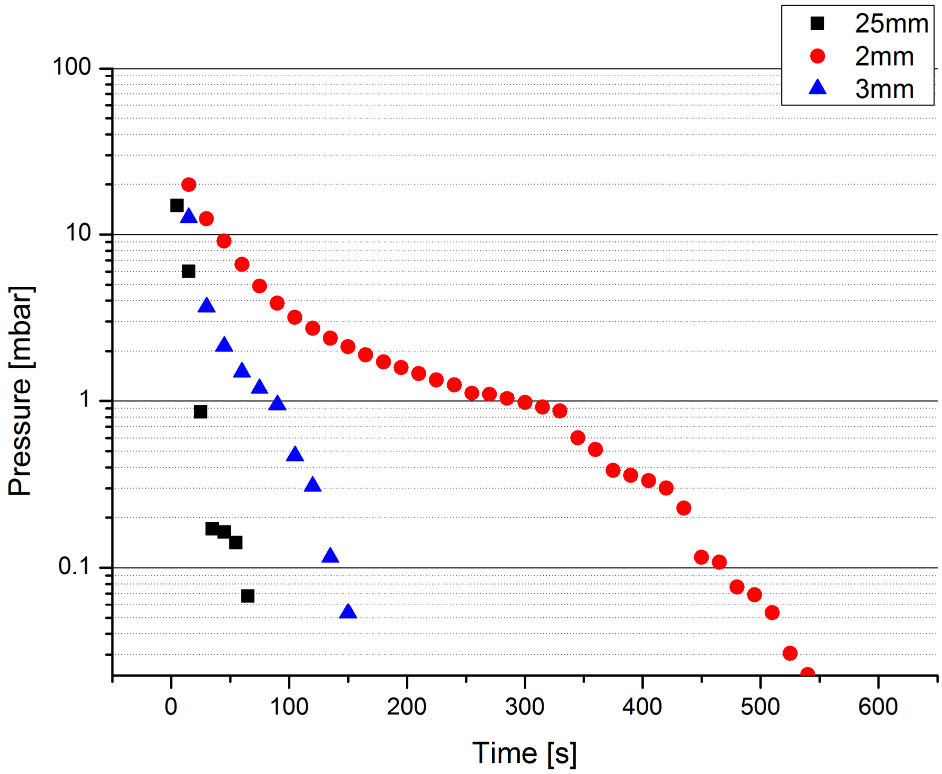


FIGURE 4: MEASUREMENT OF THE PUMPING SPEED S

Another interesting point is the relation between the pumping time and the size of the linking tubes. This is examined in the following experiment. Firstly a normal tube with 25 mm diameter is taken to evacuate a vessel with the volume: $(3{,}0 \pm 0{,}1)l$. Over a time interval the time and the responding pressure is noted. In the second step a capillary with 2 mm diameter and 9,5 cm length is added (see figure 5) and the time is measured again. For the third time a further capillary with 3 mm diameter and 9,5 cm length is added again on top of the big tube and the second capillary.

To examine the data the pressure is plotted as a function of the pumping time for the three cases by using a semi-log scale.



GRAPH 1: T – P – DIAGRAM, DATA ON PAGE: 12F.

The linear fit at low pressure gives an equation for p(t): (p(t) = y, t = x)

$$p(t) = p_0 \cdot \exp\left(-\frac{S}{V} \cdot t\right) \quad (6)$$

Experimental evaluation:

For the tube with 25 mm diameter one obtains for S at the pressure 0,3 hPa:

$\frac{S_{eff}}{V} = 0{,}005\,\frac{1}{s}$; (see function of the green curve in graph 4)

The volume of the vessel is $(3{,}0 \pm 0{,}1)\ l$.

➔ $S_{eff} = (0{,}054\ \pm\ 0)\,\frac{m^3}{h}$

For the tube and the 2 mm capillary at a pressure of 0,3 hPa S_{eff}:

$S_{eff} = 0{,}004\,\frac{1}{s} \cdot 0{,}003\ m^3$; (see function of the blue curve in graph 4)

➔ $S_{eff} = (0{,}043 \pm 0)\,\frac{m^3}{h}$

The effective pumping speed at a pressure of 5 hPa could not be calculated as the formulas in the graph 4 just count for pressures below 1 hPa and there wasn't enough data to obtain a formula for pressures above 1 hPa. Although the effective pumping speed is calculated theoretically below.

For the tube, the 2 mm and the 3 mm capillary at a pressure of 0,3 hPa:

$S_{eff} = 0{,}003 \frac{1}{s} \cdot 0{,}003\ m^3;$

➔ $S_{eff} = (0{,}032 \pm 0) \frac{m^2}{h}$

4. SUMMARY

The experiment clearly showed the relations of pressure and pumping speed.
In case we want to construct a vacuum pump by ourselves we should put great emphasis on the size of the tubes. As seen in the last experiment the diameter of the tubes is very important for the pumping time and the reached pressure (it counts with the power of 3 or even 4).

5. QUESTIONS

1. *What is an ideal gas? Why are you allowed to treat air as an ideal gas for the pressure of interest here?*

An ideal gas consists of non extended spheres which only interact with each other by elastic collisions. Other interactions such as Van-der-Waals and electrostatic forces do not occur.
At our range of pressure: 0,02 mbar – 1 mbar, the air can be seen as an ideal gas, as the molecules only interact with each other when their distance becomes quite small. At low pressure their mean distance is big enough (0,2 cm - 8 cm), so no interactions take place.

2. *Explain the thermal conductivity of a gas!*

The thermal conductivity of a gas describes the ability of a gas to conduct heat.
The conduction is done by elastic collisions; one molecule hits another and transfuses parts of its kinetic energy on the other molecule. Thus, according to the equation $\Delta E_{kin} = \frac{1}{2} m \Delta v^2 = \frac{3}{2} k_B \Delta \mathrm{T}$, the temperature of the hit object increases.
The actual conductivity of a gas between two objects is measured by the transferred energy. The gas molecule hitting the warmer object receives a bit of energy (kinetic energy) and gets reflected with a higher velocity. Then if the molecule hits the colder object it transfers some of its energy and the object gets warmer as its molecules swing faster.

3. *If the thermal conductivity of a gas is independent of pressure, why is the envelope of a thermos bottle evacuated?*

At first it is necessary to say that the conductivity at very low pressures (molecular flow, see below) is dependent on the pressure. Then the conductivity is proportional to the density which is proportional to the pressure. Thus the thermosflask's envelop isolates the bottle better if it is evacuated as the conductivity is lower. Furthermore the amount of the elastic collisions is reduced as there are fewer molecules within the envelope. Consequently the heat cannot be transported as effective and the temperature within the bottle changes much slower.

4. *What is the order of magnitude of the thermal conductivities of copper, water, air, concrete? What are the practical consequences?*

Substance	Thermal conductivity $\left[\frac{W}{m\cdot K}\right]$
Copper	470
Air	0,025
Water	0,6
Concrete	0,8-1,3

Consequently copper seems to conduct the heat very well and can be used as a conductor, whereas the other materials are isolators or accumulators. For example buildings are made with concrete to isolate the building.

5. *What is the meaning of molecular flow?*

Molecular flow exists only at very low pressures (approx. < 0,5 mbar). In this case there are only a few molecules within a gas. The exact definition of molecular flow is, that the mean free path must be larger than the tested object itself, consequently the molecules hit the walls more often than they hit each other.

6. *You are trying to evacuate a vessel through a 1 mm capillary. What pressure can you expect after 10 minutes of pumping?*

After 10 minutes of pumping you should still be in the region where the gas behaves laminar.

$\rightarrow L_{1mm} = \frac{\pi \cdot d^4}{128 \cdot \eta \cdot l} \cdot \bar{p} \approx \frac{\pi \cdot d^4}{128 \cdot \eta \cdot l} \cdot p(t) = a \cdot p(t), \quad$ with $a = \frac{\pi \cdot d^4}{128 \cdot \eta \cdot l} = 1{,}42 \cdot 10^{-8} \frac{m^4}{kg \cdot s}$

With $S_{1mm} = \frac{1}{\frac{1}{S} + \frac{1}{L_{1mm}}} = \frac{S \cdot a \cdot p(t)}{S + a \cdot p(t)}$ and using formula (6) one gets

$$p(t) = p_0 \cdot \exp\left(-\frac{S \cdot a \cdot p(t)}{V \cdot (S + a \cdot p(t))} \cdot t\right)$$

This formula is solved numerically with mathematica:

$$\mathrm{FindRoot}[p == 100000 * \mathrm{Exp}[-3.7/3600 * 1.42 * 10\text{^}(-8) * p * 600/(0.003 * (3.7/3600 + 1.42 * 10\text{^}(-9) * p))], \{p, 1\}]$$

➔ $\boldsymbol{p\ (10\mathrm{min}) = 14{,}85\ mbar}$

7. *The pressures reached with a commercial UHV arrangement are typically around $4 \cdot 10^{-11}\ hPa$. Calculate the mean free path λ at this pressure.*

$\lambda = \frac{1}{\sqrt{32}\rho F}, \quad \rho = density, F = cross - sectional\ area$

By using $\rho = \frac{N}{V} = \frac{p}{k_B \cdot T}$, one gets

$$\lambda = \frac{k_B \cdot T}{\sqrt{32} \cdot F \cdot p} = see\ table$$

F for an air molecule can be calculated by the mean molecule-radius which is around

$r = 3{,}7 \cdot 10^{-10} m$. Thus $F = \pi r^2 = 4{,}3 \cdot 10^{-19}\ m^2$

Parameter	**Value**
$\boldsymbol{k_B} \left[\frac{J}{K}\right]$	$1{,}380658 \cdot 10^{-23}$
T [K]	293,16
F $[\boldsymbol{m^2}]$	$4{,}3 \cdot 10^{-19}$
p $[\boldsymbol{hPa}]$	$4 \cdot 10^{-11}$
$\boldsymbol{\lambda}$ **[m]**	$\mathbf{415994}\ \boldsymbol{m}$

6. Attachement

- 1st measurement: Calibration of the Pirani gauge:

p [mbar]	I [mA]
0,028	3,8
0,038	4
0,089	5
0,15	5,2
0,51	9
0,64	10
1	14,7
1,8	27,4
3,1	34
5	40,4
11,1	47,5
17,6	50,2
29,8	52,4
40,8	53,4
109,7	55,1
153,3	55,4
280,3	55,7
388,4	55,8
505,8	55,8
739,4	56
960	56,1

- 2nd measurement: Pumping speed:

	First trial	**Second trial**	**Third trial**
V [ml]	Time [s]	Time [s]	Time [s]
90	23	25	21
80	43	46	41
70	65	66	65
60	85	87	82
50	105	106	103
40	126	128	124
30	144	148	144
20	166	168	165

- 3rd measurement: Effective pumping speed:

tube 25 mm diameter		tube + 2 mm capillary		tube +3mm capillary	
t [s]	I [mA]	t [s]	I [mA]	t [s]	I [mA]
5	53,2	15	53,8	15	52,5
15	44,8	30	52,4	30	36,5
25	16,1	45	50,1	45	27,3
35	5,1	60	46,2	60	22,2
45	4,1	75	41,5	75	19,4
55	3,8	90	37,4	90	17
65	3,7	105	33,9	105	11,8
75	3,7	120	31,3	120	9,9
85	3,7	135	29,1	135	7,5
95	3,7	150	27,2	150	6,7
105	3,7	165	25,5	165	6,2
115	3,7	180	24,1	180	5,5
120	3,7	195	23	195	5,3
		210	21,9	210	5,3
		225	20,8	225	5,3
		240	20	240	5,3
		255	18,7	255	5,3
		270	18,5	270	5,2
		285	17,9	285	5,2
		300	17,3	300	5,2
		315	16,7	315	5
		330	16,2	330	5,1
		345	13,3	345	5
		360	12,3		
		375	10,8		
		390	10,5		
		405	10,2		
		420	9,8		
		435	8,9		
		450	7,5		
		465	7,4		
		480	7		
		495	6,9		
		510	6,7		
		525	6,4		
		540	6,3		
		555	6,1		
		570	5,5		
		585	5,4		
		600	5,4		

7. SOURCES

1. M. Saß. “Umgang mit Unsicherheiten”. 09.07.08.
2. M. Saß. “Thermodynamics – Vacuum“. 27.11.08.

Equipment data:

- Experimental setup 1
- Voltcraft VC44
- Pump: Alcatel 2004A
- Pressure measure instrument: ILMVAC